

Practice Exam 3

1. Determine whether the following is a subspace of \mathbb{R}^4 . If it is, find a basis and the dimension.

$$\text{a) } W = \left\{ \begin{bmatrix} a \\ 2a+b \\ 0 \\ b+2 \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\text{b) } W = \left\{ \begin{bmatrix} a+2b-c \\ 2a+3c \\ b-c \\ b+2c \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

$$\text{c) } W = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} : \begin{array}{l} 2a-3b+c=0 \\ a-4d=0 \\ b+c+d=0 \end{array} \right\}$$

2. Let $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

a) Is $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ in $\text{Col}(A)$?

b) Is $\begin{bmatrix} 2 \\ -4 \\ 7 \\ 5 \\ 0 \end{bmatrix}$ in $\text{Nul}(A)$?

c) Find the dimension of $\text{Col}(A)$.

d) Find the dimension of $\text{Nul}(A)$.

e) Find a basis for $\text{Col}(A)$.

f) Find a basis for $\text{Nul}(A)$.

g) Find a basis for $\text{Row}(A)$.

3. Determine whether the polynomials $1, 1-t+3t^2, 2-4t+t^2, 6-18t+9t^2-t^3$ form a basis for \mathbb{P}_3 .

4. a) Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ relative to the basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$

$$\mathbf{b}_1 = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 3 \\ -6 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

- b) Find the vector \mathbf{v} if

$$[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

5) Let $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 5 \\ -22 \end{bmatrix}$$

a) Find the dimension of W .

b) Find a basis for W .

c) Do $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span \mathbb{R}^3 ? Justify your answer.

6) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for a vector space V , where

$$\mathbf{b}_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \quad \mathbf{c}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix} \quad \mathbf{c}_2 = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

a) Find the change of basis matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$.

b) If $\mathbf{v}_{\mathcal{B}} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$, find $\mathbf{v}_{\mathcal{C}}$.